Scientific Monthly, volume 8, no. 3, March, 1919: "Charles Edward Pickering" and full-page portrait, 284–286.

Sewanee Review, volume 27, January–March, 1919: "Edward Kidder Graham" [1876–1918, president of the University of North Carolina], by A. Henderson, 101–106.

Transactions of the Academy of Sciences of St. Louis, volume 23; all of the mathematical articles so far published in this volume are by the septuagenarian physicist, F. E. Nipher, author of the little book *Introduction to graphical algebra* published about twenty years ago. No. 4, July, 1916: "Disturbances impressed upon the earth's magnetic field," 153–162; "Gravitation and electrical action," 163–175—No. 5, November, 1917: "Gravitational repulsion," 177–192—No. 6, May, 1918: "Graphical algebra involving functions of the nth degree," 193–204—No. 7, January, 1919: "Graphical algebra," 205–212.

Transactions of the American Mathematical Society, volume 20, no. 1, January, 1919: Portrait frontispiece and notice of Maxime Bôcher; [This portrait may be obtained by sending twenty cents in postage stamps to the Society, 501 West 116th Street, New York City.] "Necessary conditions in the problems of Mayer in the calculus of variations" by G. A. Larew, 1–22; "Linear equations with unsymmetric systems of coefficients" by Anna J. Pell, 23–39; "On convex functions" by H. Blumberg, 40–44; "Projective transformations in function space" by L. L. Dines, 45–65; "On the order of primitive groups (IV)" by W. A. Manning, 66–78.

AMERICAN DOCTORAL DISSERTATIONS.

G. S. Counts, *Arithmetic tests and studies in the psychology of arithmetic* (Supplementary Educational Monographs, vol. 1, no. 4). University of Chicago Press, Chicago, 1917. 4 + 127 pp. (Chicago, 1916.)

G. James, 1882– , *Some theorems on the summation of divergent series*. New York, 227 West 17th St., W. D. Gray, 1917. 28 pp. (Columbia, 1917.)

W. S. Monroe, *Development of arithmetic as a school subject*. (Reprinted from *United States Bureau of Education*, Bulletin, 1917, no. 10.) Washington, D. C., 1917. 170 pp. (Chicago, 1915.)

Agnes L. Rogers, *Experimental tests of mathematical ability and their prognostic value*. (Teachers College, Columbia University. *Contributions to Education*, no. 89.) New York, Columbia University, 1918. 5 + 118 pp. (Columbia University, 1917.)

C. H. Yeaton, 1886– , *Surfaces characterized by certain special properties of their directrix congruences*. (Reprinted from *Annali di matematica*, series 3, vol. 26, 1916.) 3 + 33 pp. (Chicago, 1915.)

UNDERGRADUATE MATHEMATICS CLUBS.

Edited by U. G. Mitchell, University of Kansas, Lawrence.

CLUB ACTIVITIES.

The Mathematics Club of the University of Kansas, Lawrence, Kansas. [1918, 35–36, 450–1, 459.]

The officers of the club for the year 1918–19 are as follows: President, Wealthy Babcock '19; vice-president, Faye Dodderidge '19; secretary-treasurer, Edith Whitcher '19; reporter, Viola Engle '19; faculty adviser, Professor Charles H. Ashton; program committee, Rachel Bell '19, Viola Engle '19, Faye Dodderidge '19, Josephine Montague '19.

Below are given the programs for the winter and spring quarters.

December 30, 1918: "Chief contributions of mathematics to astronomy" by Professor Ellis B. Stouffer.

January 13, 1919: "The slide rule" by Evelina Watt '20.
January 27: "Valid aims of teaching mathematics in secondary schools"[1] by Josephine Montague '19.
February 10: "Russian peasant method of calculation" by Viola Engle '19; "Probability curves" by Beatrice Hagen '20.
February 24: "Abridged notation" by Jessie Craig '20.
March 10: "History of the calculus" by Vesta Shafer '19.
March 24: "Trilinear coördinates" by D'Estell Tremaine '19.
April 7: "Method of reciprocal polars" by Marie Brown '19.
April 21: "Origin of logarithms" by Carroll McDowell '19; "Polar planimeter" by Ruth Kelsey '20.
May 5: "Methods of projection" by Hazel Quick '19.
May 19: "Projective reflection" by Nellie Young '19.
June 2: Annual picnic.

THE JUNIOR MATHEMATICS CLUB, University of Minnesota, Minneapolis, Minn. [1918, 312].

The only officers of the club for the current year are an executive committee consisting of Professor Raymond W. Brink, chairman, Miss Ella Thorp, Instructor, and Laura Menk '19.

The programs given so far this year are as follows.

December 12, 1918: "Dimensionality" by Dr. Chester H. Yeaton, Instructor; "Leonhard Euler" by Ruth Stephens Gr.
February 6, 1919: "Gottfried Wilhelm Leibniz" by Lois Huney '19; "A geometrical method of summing a geometrical series" by Professor William H. Bussey.

Refreshments were served in the faculty parlor after each meeting.

TOPICS FOR CLUB PROGRAMS.

15. THE NUMBER π.

The fact that at least a dozen times in the club programs published during 1918 one finds topics relating to the number π such as "The number π," "Various definitions of π," "History of π," "Squaring the Circle" (which occurs six times), "Quadrature of the Circle," etc., is evidence that investigations of this remarkable number are still as interesting as they were in the time of Archimedes.

Since no member of a mathematics club is a "paradoxer," in De Morgan's sense of the term, little of this interest can be credited to the fact that the problem of the quadrature of the circle is, as De Morgan[2] put it, "connected with one of those propensities, the love of the marvellous, which, carried to an undue extent, tend more than others to throw the mind off its balance, and destroy the comfort

[1] *Cf.* "Valid aims and purposes for the study of mathematics in secondary schools," by A. Davis, *School Science and Mathematics*, vol. XVIII, pp. 112–123, 208–220, 313–324. (Committee Report, Mathematics Club of Chicago.)

[2] In his article "Quadrature of the Circle" in the *Penny Cyclopedia*, vol. 19, p. 186.

of the individual." In fact, the natural interest of mathematicians has undoubtedly been greatly lessened by the activities of "circle-squarers" of the type made famous by De Morgan's witty satire.

The desire for a better understanding of transcendental numbers leads the student of mathematics at the present time to a scientific interest in π, since π was the first transcendental number encountered by humanity (although not the first whose transcendence was proved) and it seems natural to think of it as the simplest and most familiar of the transcendental numbers. That π was first discovered in its relation to the circle is, of course, a mere accident. It was long since pointed out[1] that π is a number which occurs in various natural relations and would enter into analysis from whatever side the subject was approached.

When the ancient Greeks first attempted to find the length of the diagonal of a unit square, they were probably of the opinion that such a fraction (greater than unity, of course) existed, but that they were merely unable to determine it. Undoubtedly many a Greek mathematician had spent much time in fruitless testing of fractions, hoping to find one whose square was 2, before the suspicion arose that no such fraction existed and some one finally succeeded in making the proof to that effect as given by Euclid. Even then they did not conceive of a new number, different in kind from any they knew but no less definite, and invent a symbol to represent it.

Of a somewhat similar nature was the problem of determining the number which should represent the ratio of a circle to its diameter. The problem was, however, much more difficult, for two reasons. In the first place, it was much more difficult to test whether or not a given number was greater or less than the desired number and, in the second place, they were unable to prove that no such rational number existed. It must have been many centuries after the first approximate values were determined before any one suspected that there might be a number, definite and exact, but entirely different in nature from any integer or fraction, which represented the ratio of a circumference to its diameter. It was many more centuries before the first definite information regarding the true nature of π was established when J. H. Lambert, in 1761, communicated to the Berlin Academy an essentially rigorous proof[2] of the irrationality of π. Even

[1] *Cf.* DeMorgan, *Budget of Paradoxes* (London, 1872), pp. 171–172, second ed., edited by D. E. Smith (Chicago, 1915), vol. 1, pp. 284–286; also, Ball, *Mathematical Recreations and Essays*, 4th ed. (London, 1905), pp. 249–50, 5th ed. (London, 1911), p. 295.

[2] The usual citation for Lambert's proof is his "Mémoire sur quelques propriétés remarquables des quantités transcendentes circulaires et logarithmiques," *Mémoires de l'Académie de Berlin* for 1761, Berlin, 1768, pp. 265–322. It is also cited as published in *Beiträge zum Gebrauche der Mathematik*, Bd. II, Berlin, 1770, S. 140–149.

The following paragraph from E. W. Hobson's *Squaring the Circle*, Cambridge, 1913, p. 44, is worthy of note in this connection.

"It has frequently been stated that the first rigorous proof of Lambert's results is due to Legendre (1752–1833), who proved these theorems in his *Eléments de Géométrie* (1794), by the same method, and added a proof that π^2 is an irrational number. The essential rigour of Lambert's proof has however been pointed out by Pringsheim (*Münch. Akad. Ber.*, Kl. 28, 1898), who has supplemented the investigation in respect of the convergence."

before Lambert's proof was given men were coming to believe that π was not only irrational but not an algebraic irrational. This belief was rendered the more probable by Liouville's proof[1] in 1840 of the existence of transcendental numbers, and finally confirmed by Lindemann's proof[2] in 1882 of the transcendence of π.

The literature of the subject is abundant and steadily increasing. For various articles in the periodical literature the reader will, of course, consult the mathematical encyclopedias, the Royal Society Index (especially pp. 233 and 434–436) and the volumes of the International Catalogue. Among the best recent special discussions of the subject are those of Hobson,[3] Young,[4] Beman and Smith,[5] Ball,[6] Enriques,[7] Teixeira,[8] Rudio,[9] Schubert,[10] and Tropfke.[11]

In the books of Beman and Smith, Young and Hobson the development of the subject is considered as falling into three periods, the last named author devoting a chapter to each period.

The first period may be characterized as the *empirical* period, extending from the earliest attempts at the quadrature of the circle to the invention of the calculus in the second half of the seventeenth century. During this period approximations for π are obtained by purely geometrical means until the limit of refinement of that method is reached.

The second period may be characterized as the *analytic* period, extending from the invention of the calculus about a century to the proof of the irrationality of π by Lambert in 1761. During this period, by means of the more powerful methods of the new analysis, π is expressed in terms of infinite products,

[1] The simpler of Liouville's methods is given by Hobson, *loc. cit.*, pp. 44–46.

[2] *Mathematische Annalen*, Bd. 20, S. 220–224, and *Berichte der Berliner Akademie*, 1882, Bd. 2, S. 679–682. Lindemann's proof is based upon Hermite's proof (*Comptes Rendus*, vol. 77, pp. 18–24, 74–79, 226–233, 285–292) of the transcendence of e.

Simplified forms of Hermite's and Lindemann's proofs are given in the books by J. W. A. Young (pp. 402–416) and Beman and Smith (pp. 61–67) cited below, and in various other books readily accessible.

[3] E. W. Hobson, *Squaring the Circle*, Cambridge University Press, 1913.

[4] J. W. A. Young, *Monographs on Modern Mathematics*, New York, 1911, Monograph IX, "The History and Transcendence of π," by D. E. Smith..

[5] W. W. Beman and D. E. Smith, *Famous Problems of Elementary Geometry*, Boston, 1897, English translation of Klein's *Vorträge uber ausgewählte Fragen der Elementar-Geometrie*, Leipzig, 1895, pp. 54–80.

[6] W. W. R. Ball, *Mathematical Recreations and Essays*, 5th ed., London, 1911, pp. 293–306; 4th ed., London, 1905, pp. 247–261.

[7] F. Enriques, *Fragen der Elementargeometrie*, Deutsche Ausgabe von H. Fleischer, II Teil, Leipzig, 1907.

[8] F. G. Teixeira, *Sur les problèmes célèbres de la géométrie élémentaire non résolubles avec la règle et le compas*, Coïmbre, 1915, pp. 83–104.

[9] F. Rudio, *Archimedes, Huygens, Lambert, Legendre. Vier Abhandlungen über die Kreismessung*, Leipzig, 1892. See also articles "Der Bericht des Simplicius uber die Quadraturen des Antiphon und des Hippokrates" and "Zur Rehabilitation des Simplicius" by Rudio in *Bibliotheca Mathematica*, 1902, pp. 7–62, and 1903, pp. 13–18, 1907, pp. 13–22. Also a discussion of Rudio's first article by P. Tannery in the same volume (Bib. Math. for 1902), pp. 342–349.

[10] H. Schubert, *Mathematical Essays and Recreations*, translated by T. J. McCormack, Chicago, 1910, pp. 112–143.

[11] J. Tropfke, *Geschichte der Elementar-Mathematik*, Bd. 2, Leipzig, 1903, S. 108–138.

continued fractions and infinite series. Approximations[1] are obtained far beyond any conceivable practical need and mathematicians come to suspect that π is not only irrational but not an algebraic irrational.

The third period may be called the *critical* period, since it was devoted to "critical investigations of the true nature of the number π itself, considered independently of mere analytical representation." [2] The period extends from the middle of the eighteenth century late into the nineteenth century when the transcendence of π is finally definitely established.

William Jones[3] seems to have been the first to make use of the symbol π with its present special significance, but its permanent use as such was chiefly due to the influence of Euler.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

Send all communications about Problems to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

2773. Proposed by JOSEPH ROSENBAUM, Milford, Conn.

Point out the fallacy in the proof following the problem:

In the triangle $A_1B_1C_1$ let M be a point such that the sum of the distances from it to the sides is a maximum; also, let $A_2B_2C_2$ be a triangle formed by drawing lines through the vertices A_1, B_1, and C_1 parallel to their opposite sides. Then the sum of the distances from M to the sides of the triangle $A_2B_2C_2$ is a minimum.

Proof.—Because the sides of the two triangles are parallel in pairs, the sum of the distances from a variable point P in triangle $A_1B_1C_1$ to the six sides of the two triangles is constant. Now by hypothesis M is a point for which one part of this constant sum is a maximum, and hence it follows that the other part is a minimum.

2774. Proposed by FRANK IRWIN, University of California.

Evaluate the circulants

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ n & 1 & 2 & \cdots & n-2 & n-1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 2 & 3 & 4 & \cdots & n & 1 \end{vmatrix}, \qquad \begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & a_n \\ a_n & a_1 & a_2 & \cdots & a_{n-2} & a_{n-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_2 & a_3 & a_4 & \cdots & a_n & a_1 \end{vmatrix},$$

where, in the latter, $a_1, a_2, \cdots a_n$ form an arithmetical progression.

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[1] A condensed table of approximations of π as determined by various men is given by J. W. L. Glaisher, *Messenger of Mathematics*, vol. 2, pp. 122–128, and in vol. 3, pp. 45–46, some corrections are made of the table given on p. 122 of vol. 2. The approximation was carried to 707 places by William Shanks in 1873 (*Proceedings of the Royal Society of London*, vol. 21, p. 318 and vol. 22, p. 45). A considerable list of approximations is given by Ball, *loc. cit.*, 4th ed., pp. 250–261, 5th ed., pp. 296–306.

[2] Hobson, *l. c.*, p. 12.

[3] *Synopsis Palmariorum Matheseos*, London, 1706, pp. 243, 263, *et seq.* Cited by Ball, *loc. cit.*, 4th ed., p. 250, 5th ed., p. 296, and by others. Concerning its early use by others, see article "Sur le premier emploi du symbole π pour 3.14159 $\cdots$" by G. Eneström, *Bibliotheca Mathematica*, 1889, p. 28.